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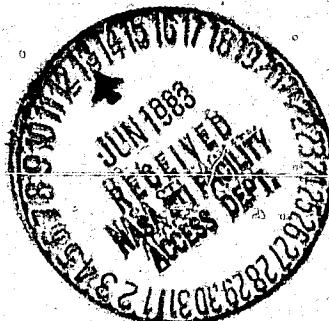
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# Pivoting and Slip in an Angular Contact Bearing

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PIVOTING AND SLIP IN AN ANGULAR CONTACT BEARING

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ABSTRACT

Pivoting slips are calculated for the ball-race and ball-ball contacts in a retainerless bearing. The calculation is kinematic, ignoring all inertial loadings. Pure spin and uniform precession of the balls are considered. Pivoting slip magnitudes are compared with several other kinds of slip which have previously been reported in an R4 size bearing.

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INTRODUCTION

To avoid confusion a distinction between spin and pivoting is made:

Spin: That component of the angular velocity of a single body along its axis of figure;

Pivoting: That component of the relative angular velocity of two bodies in contact which is normal to their plane of tangency.

Pivoting is a relative angular velocity arising in ball bearings because, for example, ball and race spins are not both parallel to the Hertz

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contact area. Pivoting slip is the associated relative surface velocity, maximum at the edge of the Hertz area and decreasing to zero at its center.

Pivoting has been of concern for unlubricated bearings because it was thought that dry friction would concentrate all the pivoting at either the inner or outer ball-race contact (1). Ball spin orientation was to be "controlled" by one or the other race. If control should suddenly change, an undesirable step in bearing performance would result. Since, however, most bearings are lubricated, this model is probably inappropriate to most bearing operation. A careful experimental search for the predicted performance step in a starved but lubricated instrument bearing was negative (2).

On the other hand, pivoting slip may be important in an EHL contact because of the shear strain it requires in the lubricant. A shear-activated oxidation or polymerization of organic lubricants, changing them from fluid to solid, has been suggested (3-7) as a failure process in starved ball bearings. In this paper pivoting slips are calculated for ball-race and ball-ball contacts in a retainerless bearing, for balls performing pure spin and uniform precession. The slips are then evaluated for an R4 instrument bearing, and compared with previously reported circumferential drag and transverse precessional slips in the same bearing.

#### BALL-RACE PIVOTING

The bearing is assumed to be rotating slowly enough for equal inner and outer contact angles. Consider a coordinate frame centered on a ball with OY normal to the bearing spin axis (Fig. 1).

Race angular velocities are

$$\bar{\gamma}_0 = \dot{\gamma}_0 \bar{T} \quad (1)$$

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$$\dot{\gamma}_I = \gamma_I \dot{t}$$

Ball orbit velocity is

$$\dot{\beta} = \beta \dot{t} \quad (2)$$

Ball-race contact radius vectors are

$$\bar{r}_0 = \frac{d}{2} \left[ \bar{i} \sin B + \bar{j} \cos B \right] \quad (3)$$

$$\bar{r}_I = \frac{d}{2} \left[ -\bar{i} \sin B - \bar{j} \cos B \right]$$

Ball-about-its-own-center angular velocity is

$$\dot{\delta} = (l\bar{i} + m\bar{j} + n\bar{k}) \dot{\delta} \quad (4)$$

$l$ ,  $m$ , and  $n$  are direction cosines for  $\dot{\delta}$  and may vary with time. For the angle  $\phi$  as shown in Fig. 1,  $l = \cos \phi$  and  $m = \cos(\pi/2 + \phi)$ . If  $n \neq 0$ ,  $\dot{\delta}$  is not in the plane of Fig. 1.

The inner and outer pivotings are the components of  $(\dot{\gamma}_0 - \dot{\delta})$  and  $(\dot{\gamma}_I - \dot{\delta})$  lying along  $\bar{r}_0$  and  $\bar{r}_I$ :

$$F = \frac{1}{r} \left[ \dot{\gamma} - \dot{\delta} \right] \cdot \bar{r} \quad (5)$$

Thus

$$P_0 = (\dot{\gamma}_0 - \dot{\beta} - \dot{l}\delta) \sin B - m\delta \cos B \quad (6)$$

$$P_I = (-\dot{\gamma}_I + \dot{\beta} + \dot{\delta}) \sin B + m\dot{\delta} \cos B$$

Adding Eqs. (6) together shows that "ball-race pivoting is conserved in an angular contact bearing":

$$\Sigma P = (\dot{\gamma}_0 - \dot{\gamma}_I) \sin B = S \sin B \quad (7)$$

$(\dot{\gamma}_0 - \dot{\gamma}_I) = S$  is the "total speed" of the bearing (2). This means that for all combinations of race rotations ("modes") having the same total speed, no

matter what the particular motion of  $\dot{\delta}$  might be, the total ball-race pivoting is the same.

Since  $P_0$  and  $P_I$  are components of vectors which point in opposite directions, each could separately have a large value (implying lubricant distress) while their sum remained small. Consider the absolute values  $|P_0|$  and  $|P_I|$ :

$$\Sigma |P| = |P_0| + |P_I| \quad (8)$$

$P_0$  and  $P_I$  change sign when their arguments pass through zero, ie at the values of  $\phi$  which would be calculated for inner and outer race control. Therefore

$$\Sigma |P| = S \sin B \quad (9)$$

for

$$\phi_0 < \phi < \phi_I$$

where

$$\phi_0 = B - \sin^{-1} \left[ \frac{1}{\frac{1}{r}} \left( \dot{\gamma}_0 - \dot{\beta} \right) \sin B \right] \quad (10)$$

$$\phi_I = B - \sin^{-1} \left[ \frac{1}{\frac{1}{r}} \left( \dot{\gamma}_I - \dot{\beta} \right) \sin B \right]$$

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(These equations follow from Eqs (6) by setting  $P_0$  and  $P_1 = 0$ )

It is fairly certain that most bearing operation is within this range.

Since  $\delta/S = \rho$ , the "basic speed ratio" (a characteristic number which is constant for any bearing in all modes (2)), the total ball-race pivoting normalized with respect to  $\delta$  becomes very simply

$$\Sigma P_i / \delta = \sin B / \rho, \quad (11)$$

independent of total speed, mode, and the specific behavior of  $\delta$ . In particular, if  $\delta$  should perform a uniform precession with constant coning angle  $\theta$  (as has been observed, see below), Eq. (11) still holds so long as  $\phi_0 < \theta < \phi_1$ .

#### BALL-BALL PIVOTING

The ball-ball pivoting calculation is outlined first for a uniform ball precession where  $\delta$  is time-variable. The result is then specialized for pure spin, which is a degenerate form of precession.

Precession is of interest in a ball bearing because it has been observed experimentally (4,7). Kinematic analyses of the motion are given in (6) and (8). During precession the ball is seen to spin at rate  $s$  about an axis of figure which simultaneously cones or precesses at rate  $p$  about a fixed line.  $\vec{s}$  and  $\vec{p}$  include the coning angle  $\theta$  and form oblique components of  $\delta$ . In a uniform precession  $p$ ,  $s$ , and  $\theta$  are constant; for pure spin  $s = \theta = 0$  and  $p = \dot{\delta}$ .

To calculate ball-ball pivoting the angular velocity vectors for two adjoining precessing balls are written in a common coordinate system, as is done in (8). The scalar product of their difference with the contact normal then gives the desired pivoting. For  $\theta$  small, the result is

$$P_p = 2 \frac{d}{E} [(p + s) \sin B - s\theta \cos B \cos pt], \quad (12)$$

and for pure spin

$$P_s = 2 \frac{d}{E} [\delta \sin B] \quad (13)$$

#### DISCUSSION

It is interesting to evaluate these pivotings and slips for an R4 bearing ( $d = 0.238$  cm,  $E = 1.108$  cm,  $B = 0.31$  rad ( $18^\circ$ ),  $\rho = 2.231$ ). Typical precession parameters observed in this bearing are  $s/p \approx -0.05$ ,  $(s + \gamma) \approx \delta$ ,  $\theta \approx 0.17$  rad ( $10^\circ$ ) (8,9); also several measured slips (10,11) are available for comparison.

##### (a) Ball-Race Pivoting Slip

Equation (11) evaluated for the R4, assuming a Hertz semiwidth of 0.003 cm, and normalized with respect to ball surface velocity is

$$\delta_{b-r} = \frac{a_r \varepsilon p}{d/2 \delta} = \frac{2a_R \sin B}{\rho d} = 0.003 \quad (14)$$

It will be recalled that Eqs. (11,14) hold whether or not the ball precesses.

##### (b) Ball-Ball Pivoting Slip

Equation (13) for the R4, assuming a ball-ball Hertz contact radius of 0.001 cm, and normalized with respect to ball surface velocity, is

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$$\delta_{b-b} = \frac{a_b P}{d/2 \cdot \delta} = \frac{4a_b \sin B}{E} = 0.001 \quad (15)$$

This is the maximum pivoting slip in the ball-ball contact for pure spin. Equation (12) shows that precession adds a small time variable component. Its amplitude is  $4s\theta \cos B/\delta E = 0.00003$  for the R4, and its frequency is  $\delta$ .

Table I compares these results with five slips which have either been derived or measured in an R4 bearing. Apparently it will be necessary to include ball-race pivoting slip in any discussion of lubricant degradation due to shear activation since this particular slip amplitude is comparable to several others known to be present in the Hertz zones. However, Heathcote slip (derived on the basis that when a ball rolls in a conforming groove not all points in the contact lie on the same instantaneous rolling axis) can probably be ignored since it is more than an order of magnitude smaller.

In the ball-ball contacts on the other hand, the pivoting slips are entirely negligible compared with existing circumferential slip. This contact is characterized by high slip velocity and low contact pressure, compared to the ball-race contact, which has low slip and high pressure.

Experiments (4,7) show that lubricant degradation occurs in both the high-pressure low-slip and low-pressure high-slip contacts. It has been suggested (13) that the reactions go on all the time in every ball bearing, but that if the high pressure EHL films becomes too thin in a starved bearing, the rates of reaction greatly increase, leading to a "catastrophic failure." On the other hand, in the low pressure contacts, the lubricant degradation reactions form organic polymer in situ on the ball tracks. This changes the ball-ball contact from steel-steel to polymer-polymer, and may

be the source of the load-carrying capacity which is observed in retainerless bearings (6). It might be said that a full complement bearing self-constructs an ultra light-weight, geometrically perfect, plastic ball separator in just the right location by this slip degradation process.

#### NOMENCLATURE

a      Hertz semi width, cm  
B      contact angle, rad  
d      ball diameter, cm  
E      pitch diameter, cm

$\begin{matrix} \mathbf{i} \\ \mathbf{j} \\ \mathbf{k} \end{matrix}$       unit vectors

$\begin{matrix} \mathbf{l} \\ \mathbf{m} \\ \mathbf{n} \end{matrix}$       direction cosines

P      pivoting

$\dot{\mathbf{p}}$       precession component of  $\dot{\gamma}$ , magnitude  $\dot{p}$ , Hz

$\dot{\mathbf{r}}$       radius vector, magnitude  $\dot{r}$ , cm

S      total speed, Hz

$\dot{\mathbf{s}}$       spin component of  $\dot{\gamma}$ , magnitude  $\dot{s}$ , Hz

$\dot{\mathbf{s}}$       normalized pivoting slip

$\dot{\mathbf{\beta}}$       ball orbit angular velocity, magnitude  $\dot{\beta}$ , Hz

$\dot{\mathbf{\gamma}}$       race angular velocity, magnitude  $\dot{\gamma}$ , Hz

$\dot{\mathbf{\delta}}$       ball angular velocity, magnitude  $\dot{\delta}$ , Hz

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$\theta$  coning angle, rad  
 $\rho$  basic speed ratio  
 $\varphi$  angle locating  $\delta$ , rad

Subscripts:

$b$  ball  
 $I$  inner race  
 $O$  outer race  
 $p$  precession  
 $r$  race  
 $s$  spin

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REFERENCES

1. Hamrock, B.J., and Dowson, D., Ball Bearing Lubrication, John Wiley, New York, 1981.
2. Kingsbury, E. P., "Basic Speed Ratio of an Angular Contact Bearing," ASME Journal of Lubrication Technology, Vol. 102, No. 3, July 1980, pp. 391-394.
3. Walker, D. L., Sanborn, D. M., and Winer, W. O., "Molecular Degradation of Lubricants in Sliding Elastohydrodynamic Contacts," ASME Journal of Lubrication Technology, Vol. 97, No. 3, July 1975, pp. 390-397.
4. Kingsbury, E. P., "Lubricant Breakdown in Instrument Ball Bearings," ASME Journal of Lubrication Technology, Vol. 100, No. 3, July 1978, pp. 386-394.
5. Kingsbury, E. P., "Ball Motion Perturbation in an Angular Contact Ball Bearing," ASLE Transactions, Vol. 25, No. 3, July 1982, pp. 279-282.
6. Kingsbury, E. P., "Ball-Ball Load Carrying Capacity in Retainerless Angular Contact Bearings," ASME Journal of Lubrication Technology, Vol. 104, No. 3, July 1982, pp. 327-329.
7. Kingsbury, E. P., "Ball Contact Locus in an Angular Contact Bearing," ASME Journal of Lubrication Technology, Vol. 105, No. 2, Apr. 1983, pp. 166-170.
8. Kingsbury, E. P., "Ball Motion in Angular Contact Bearings," Wear, Vol. 11, 1968, pp. 41-50.
9. Kingsbury, E. P., "Precessional Slip in an Angular Contact Ball Bearing," Wear, Vol. 77, 1982, pp. 105-114.

10. Kingsbury, E.P., "Dynamic and Coupling Influences on Basic Speed Ratio of an Angular Contact Bearing," Wear, Vol. 63, 1980, pp. 189-196.
11. Kingsbury, E. P., "Slip Measurement in an Angular Contact Ball Bearing," ASME Journal of Lubrication Technology, Vol. 105, No. 2, Apr. 1983, pp. 162-165.
12. Heathcote, H. L., Mechanical World, Vol. 70, 1921, pp. 79.
13. Kingsbury, E., "Lubricant Breakdown Acceleration in Instrument Ball Bearings," presented at the 5th Inertial Guidance Community Contamination Control Seminar, El Segundo, California, October 16-18 1979.

TABLE I. - VARIOUS SLIPS COMPARED<sup>a</sup>

Slip Type	Reference	Formula	For R4
<u>Ball-Race</u>			
Pivoting	Present	$2a_r \sin B/\rho d$	0.003
Orbit drag <sup>b</sup>	(10)	Measured <sup>c</sup>	.004 to 0.001
Spin drag <sup>b</sup>	(11)	Measured <sup>c</sup>	.002 to 0.001
Transverse Precessional	(9)	$se/\delta \sin pt$	.008
Heathcote	(12)	$a_r^2/2d^2$	.00008
<u>Ball-Ball</u>			
Pivoting	Present	$4a_b/E \sin B$	.001
Circumferential	(9)	$2(1 - d^2/E^2 \sin^2 B)^{1/2}$	1.996

<sup>a</sup>All slips are normalized with respect to ball surface velocity.

<sup>b</sup>Orbit drag slip results from the dissipative torque due to ball group orbit motion, spin slip results separately from the dissipative torque due to ball spin.

<sup>c</sup>Measured values vary with oil type and quantity, with total speed and mode, and with the thrust load on the bearing.

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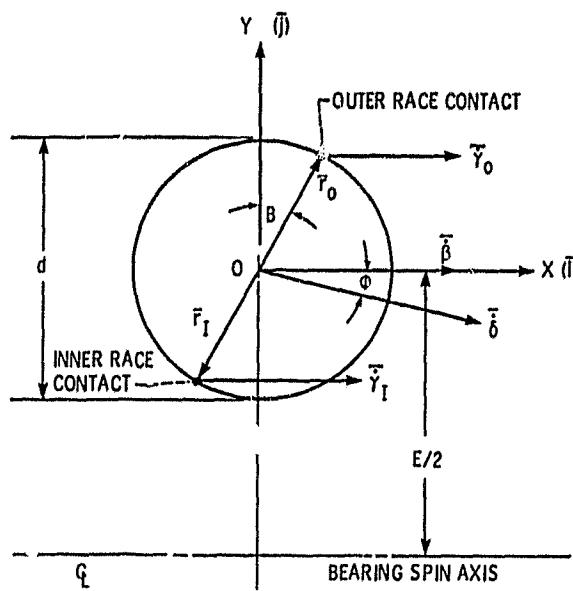


Figure 1. - Ball and race kinematics.